Period 1, May 5, 2025

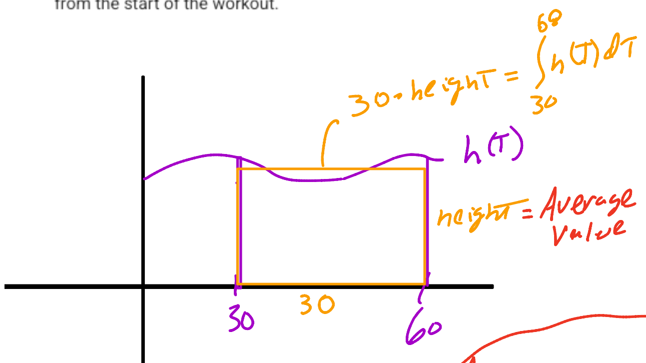
$h(T) = T^2 - 16T^{-1} + 15$

The height of an object at time $t \geq 1$ is given by $h(t) = t^2 - \frac{16}{t} + 15$. What is the velocity of the object at time $t = 3$?

$V(T) = h'(T) = 2T + \frac{16}{T^2}$

$V(3) = 2 \cdot 3 + \frac{16}{3^2} = 6 + \frac{16}{9} = \frac{54}{9} + \frac{16}{9} = \frac{70}{9} = 7.7\overline{7}$

Tara's heart rate during a workout is modeled by the differentiable function h, where $h(t)$ is measured in beats per minute and t is measured in minutes from the start of the workout.



height of Rectangle

Which of the following expressions gives Tara's average heart rate from $t = 30$ to $t = 60$?

$\frac{1}{60-30} \int_{30}^{60} h(T)\,dT$

$\int_{30}^{60} h(T)\,dT$ = Area under curve = Total # of heartbeats

Average Value = $\dfrac{\int_{30}^{60} h(T)\,dT}{30}$

Let g be the function with first derivative $g'(x) = \sqrt{x^3 + x}$ for $x > 0$. If $g(2) = -7$, what is the value of $g(5)$?

$\int_2^5 g'(x)\,dx$ = Change in Value From 2 To 5

$\int_2^5 \sqrt{x^3 + x}\, dx = 20.899159372$

$g(5) = \underbrace{g(2)}_{\text{Start}} + \underbrace{\int_2^5 g'(x)\,dx}_{\text{Change}}$

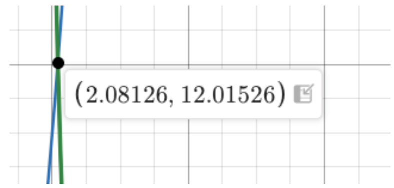
$-7 + 20.899 = 13.899$

$$f(x) = \begin{cases} k^3 + x & \text{for } x < 3 \\ \frac{16}{k^2 - x} & \text{for } x \geq 3 \end{cases}$$

Let f be the function defined above, where k is a positive constant.

For what value of k, if any, is f continuous?

$$k^3 + 3 = \frac{16}{k^2 - 3}$$



2 $y = x^3 + 3$

3 $y = \dfrac{16}{x^2 - 3}$

$F(x) = \int g(x)\,dx = \int (3 - \sqrt{x^2 + x + 4\cos x})\,dx$

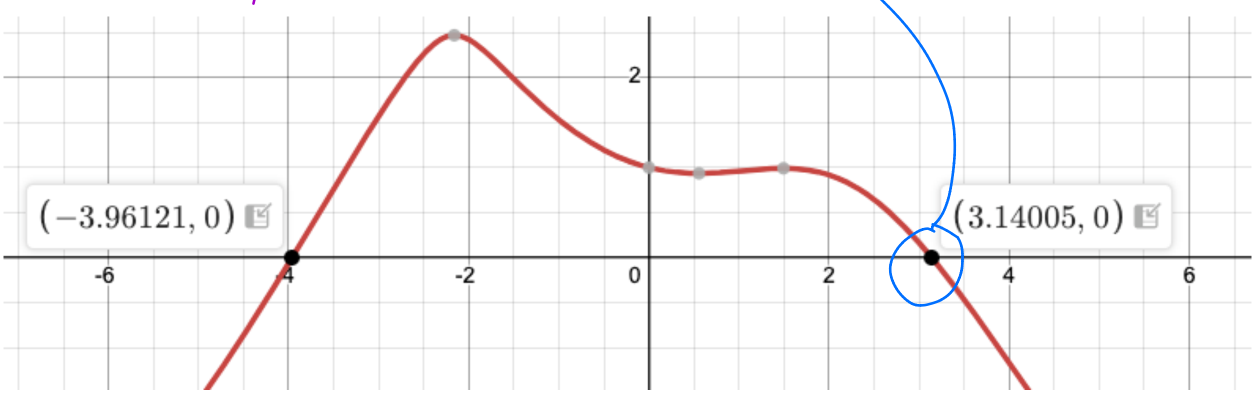
From + To negative

The function f is an antiderivative of the function g defined by $g(x) = 3 - \sqrt{x^2 + x + 4\cos x}$. Which of the following is the x-coordinate of the location of a local maximum for the graph of $y = f(x)$?

$F'(x) = 0$ or $\phi = 3 - \sqrt{x^2 + x + 4\cos x}$

$x = 3.14$

$x = \pi$



At time t, $0 < t < 2$, the velocity of a particle moving along the x-axis is given by $v(t) = t\sin(t^3)$. Let $t = b$ be the time at which the particle changes direction from moving left to moving right.

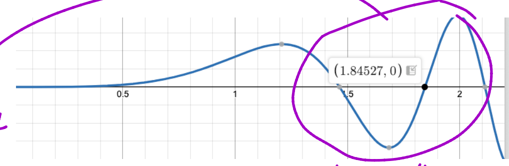
$V(T) = 0$

goes From negative To +

$$\int_0^{1.84527} |x \sin(x^3)|\, dx$$

absolute Value

What is the total distance traveled by the particle during the time interval $0 < t < b$?



$b = 1.84527$

$$y = \int_0^{1.84527} \left|x \sin x^3\right| dx$$

$= 1.01114095792$

$f'(x) = x^3 - 2x^2 + 1x - \frac{1}{2}$

Let f be the function defined by $f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{2}x$. For how many values of x in the open interval $(0, 1.565)$ is the instantaneous rate of change of f equal to the average rate of change of f on the closed interval $[0, 1.565]$?

slope

$F(0) = 0$

$$\frac{F(1.565) - F(0)}{1.565 - 0} = \frac{F(1.565)}{1.565} = \frac{-0.613569808177}{1.565} =$$

$f(1.565)$

$= -0.613569808177$

$\frac{f(1.565)}{1.565}$

$= -0.392057385417$ $= F'(x) = ?$

3 Times

average = 3

$x=2 \quad y=6$

continious

always increasing

Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(3) = 12$ and $g(5) = 18$. Which of ~~20~~, 21, and 22 are possible values for $g(6)$?

always increasing

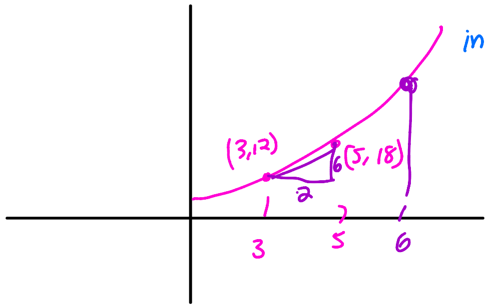
Concave up is getting bigger

Faster

Less Then 21

$18 + 3 = 21$

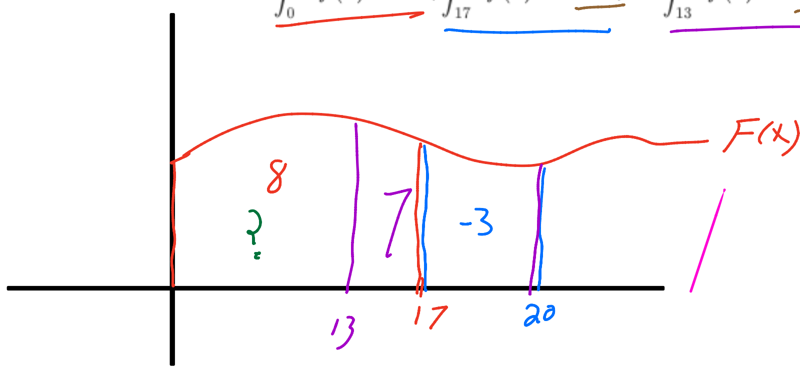
Line



$8 - 10 = -2$

$$\int_0^{13} F(x)dx = \int_0^{17} F(x)dx - \int_{13}^{17} F(x)dx$$

Let f be a continuous function such that $\int_0^{17} f(x)dx = 8$, $\int_{17}^{20} f(x)dx = -3$, and $\int_{13}^{20} f(x)dx = 7$. What is the value of $\int_0^{13} f(x)dx$?



$$\int_{13}^{17} F(x)dx = 10$$

$$\int_{13}^{17} F(x)dx + \int_{17}^{20} F(x)dx = 7$$

$$10 + -3 = 7$$

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

Ft

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Selected values of $g(t)$ are given in the table above.

(a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

$g'(x) = \text{Slope of } g(x) = F^3/\text{min}^2$

$g(1) = 15.1$

$g(5) = 20.5$

$\frac{20.5 - 15.1}{5-1} = \frac{5.4}{4} = 1.35\ F^3/\text{min}^2$

(b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

$\int_0^8 g(x)dx \approx 1(15.1) + 4(20.5) + 1(18.3) + 2(22.7)$

$\approx 160.8\ F^3$ of grain

(c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

How much grain Spoiled $= \int_0^8 w(T)dT = \int_0^8 \left(32\sqrt{\sin\frac{\pi T}{74}}\right) dT$

$\int_0^8 \left(32\sqrt{\sin\left(\frac{\pi x}{74}\right)}\right)dx$

$= 99.0514965174$

Unspoiled grain $= 160.8 - 99.051$

$= 61.749\ Ft^3$

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

unspoiled grain = Total grain − Spoiled grain

$0 >$ increasing

$0 <$ decreasing

$h(6) = 16.06317325$

adding $18.3 \ ft^3/min$

Spoiled $30 \cdot \sqrt{\sin \frac{6\pi}{74}}$

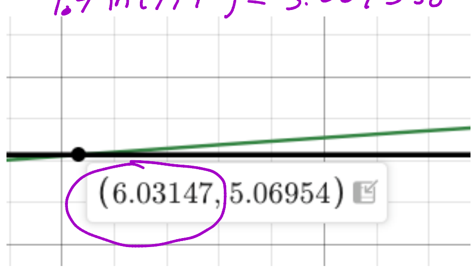
$18.3 - 16.06 =$ increasing $2.24 \ ft^3/min$

Rate of unspoiled = Rate Total − Rate of Spoiled

2. A snail is traveling along a straight path. The snail's velocity can be modeled by $v(t) = 1.4 \ln\left(1 + t^2\right)$ inches per minute for $0 \le t \le 15$ minutes.

(c) At what time t, $0 \le t \le 15$, is the snail's instantaneous velocity equal to its average velocity over the interval $0 \le t \le 15$?

$1.4 \ln(1+t^2) = 5.069538$



$\frac{1}{15-0} \int_0^{15} v(t)\,dt$

$\int_0^{15} v(t)\,dt = 76.0430738412$ Snail Travel

Average Velocity $= \frac{76.04307}{15} = 5.069538$

$v(12) = 2(12) - 1.65231 = 22.348$ in/min

$v(t) = 2t + c = 2t - 1.65231$

(d) An ant arrives at the snail's starting position at time $t = 12$ minutes and follows the snail's path. During the interval $12 \le t \le 15$ minutes, the ant travels in the same direction as the snail with a constant acceleration of 2 inches per minute per minute. The ant catches up to the snail at time $t = 15$ minutes. The ant's velocity at time $t = 12$ is B inches per minute. Find the value of B.

ant $a(t) = 2 \ in/m^2$

$v(t) = \int 2\,dt = 2t + c = 2(12) + -1.65231 = 22.348 \ in/min$

Time $0 \to 15$

$\int_{12}^{15} v(t)\,dt = \int_0^{15} v(t)\,dt = 76.0430738$

0 → 76.0430738 in → Snail T = 15

$\int_{12}^{15} (2t + c)\,dt = t^2 + ct \Big|_{12}^{15} = 76.0430738$

ant T = 12 → $v(12) = 22.348$ in/min

$15^2 + 15c - [12^2 + 12c] = 225 - 144 + 3c = 76.04307$

$81+3c=76.04307$

$c=-1.65231$

Circle

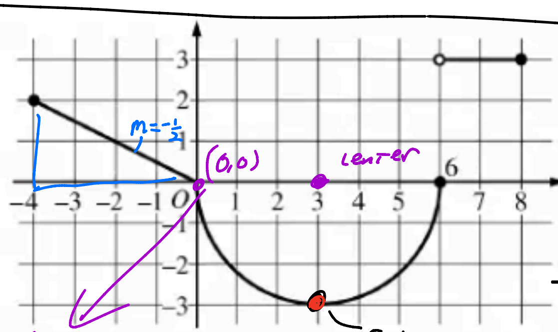
$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y-0)^2=3^2$

$x^2-6x+9+y^2=9$

Find $\frac{dy}{dx}$

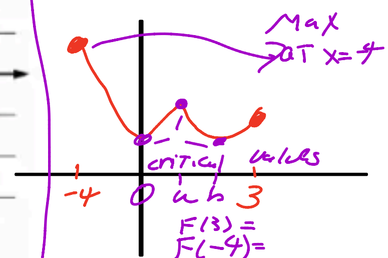
$2x-6+2y\frac{dy}{dx}=0 \Rightarrow \frac{2x-6}{-2y}=\frac{dy}{dx}=\frac{-6}{0}=\emptyset$



Graph of g

$g(3)=-3$

Example



3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)\,dt$.

(a) Find $f(7)$ and $f'(7)$.

1st step use End points

other points $F'(x)=0$ or $\emptyset$

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

x	y
-4	-10
3	6

max = 6

$g(x)$ is continious

$g(x) \neq \emptyset$

$F(x) \neq \emptyset$

$F'(x)=0$ or $\emptyset$

$F'(x)=3+g(x)$

$0=3+g(x)$ critical values

$g(x)=-3$

$X=3$

$F'(x)=0$

$F'(3)=0$

$F(-4)=3(-4)+g(-4)$

$=-12+2=-10$

$F(3)=3\cdot 3+g(3)=9+-3=6$

(c) For each of $\lim\limits_{x\to 0^-} g'(x)$ and $\lim\limits_{x\to 0^+} g'(x)$, find the value or state that it does not exist.

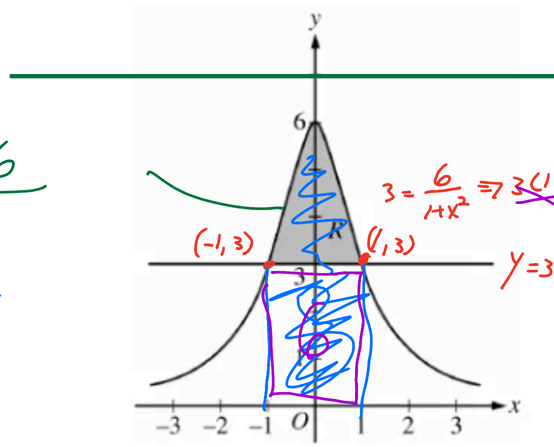
$g'(x)=$ Slope of $g(x)$

$\lim\limits_{x\to 0^-}$ Slope of $g(x)=-\frac{1}{2}$

$\lim\limits_{x\to 0^+}$ Slope of $(g(x))=\emptyset=$ DNE

$R = 3\pi - 6$

3π



$3 = \frac{6}{1+x^2} \Rightarrow 3(1+x^2) = \frac{6}{3} \Rightarrow 1+x^2 = 2$

$x^2 = 1$

$x = \pm 1$

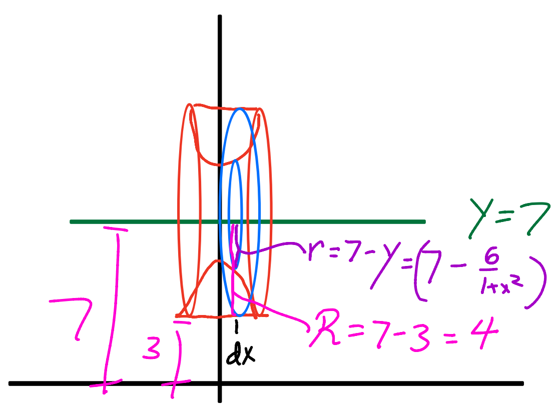
4. Let f be the function defined by $f(x) = \dfrac{6}{1+x^2}$. Let R be the shaded region bounded by the graph of f and the horizontal line $y = 3$, as shown in the figure above.

(a) Find the area of R.

$\int_{-1}^{1} \frac{6}{1+x^2}\,dx = 6\int_{-1}^{1} \frac{1}{1+x^2}\,dx = 6\arctan x \Big[_{-1}^{1} = 6\arctan 1 - 6\arctan(-1) =$

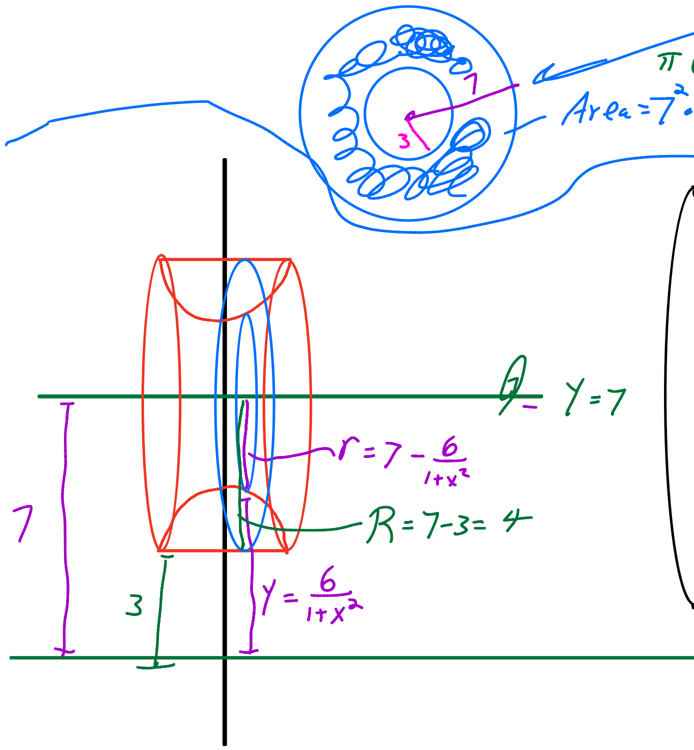
$6 \cdot \frac{\pi}{4} - 6 \cdot -\frac{\pi}{4} = 12 \cdot \frac{\pi}{4} = 3\pi$

(b) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 7$.



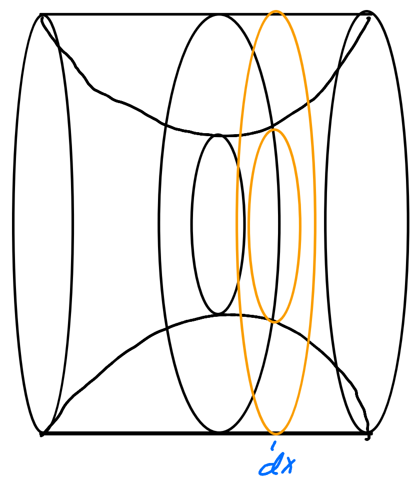
$\pi \int_{-1}^{1} [R^2 - r^2]\,dx$

$\underbrace{\pi \int_{-1}^{1} \left[4^2 - \left(7 - \frac{6}{1+x^2}\right)^2\right] dx}$



$\pi(7^2 - 3^2)$

$\text{Area} = 7^2 \cdot \pi - 3^2 \cdot \pi = 49\pi - 9\pi = 40\pi$

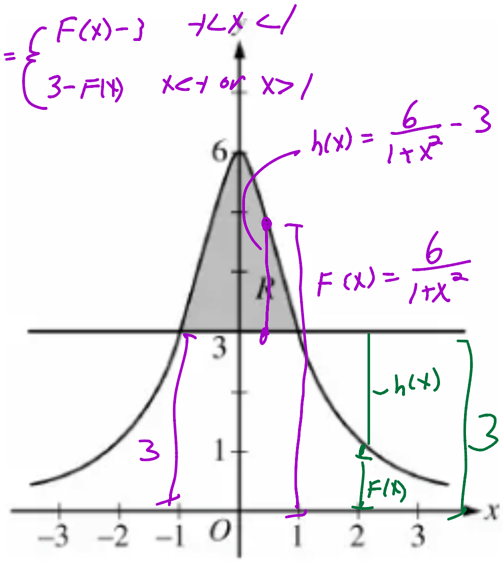


always Positive

(c) Let $h(x)$ be the vertical distance between the point $(x, f(x))$ and the horizontal line $y = 3$. Find the rate of change of $h(x)$ with respect to x at $x = 2$.

$h'(x) =$

$$h(x) = \begin{cases} F(x) - 3 & -1 < x < 1 \\ 3 - F(x) & x < -1 \text{ or } x > 1 \end{cases}$$



$h(x) = 3 - F(x)$

$h'(x) = 0 - 6(-1)(1+x^2)^{-1-1=-2}(2x)$

$h'(x) = \frac{+6}{(1+x^2)^2} \cdot 2x = \frac{+12x}{(1+x^2)^2}$

$h'(2) = \frac{+12(2)}{(1+2^2)^2} = \frac{+24}{(1+4)^2} = \frac{+24}{5^2} = \frac{+24}{25}$

$h(x) = 3 - F(x)$

5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time t seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.

$F(0) = 10$

$\underbrace{\quad}$ Rate of Change in Substance Present

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

Slope and point

$0 = (-0.02(10)^2)$ $\quad$ $(0, 10)$

$= -2 = m$ $\quad$ $T_1 = 0$

$y_1 = 10$

Line $\Rightarrow$ $y - 10 = -2(T - 0)$

$y - 10 = -2(2)$

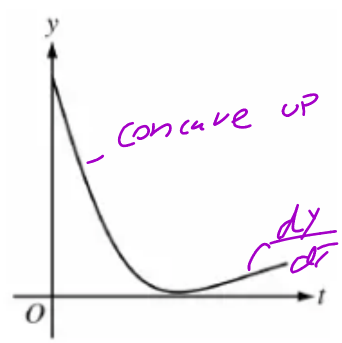
$y = 6$ grams

(b) Using the given differential equation, determine whether the graph of f could resemble the following graph. Give a reason for your answer.

$y = F(T)$

$\frac{dy}{dT} = -0.02y^2 =$ always negative

$\frac{d^2y}{dT^2} = -0.04y$



concave up

$\frac{dy}{dt} = +$

$y =$ amount of substance $=$ it must be positive

$\frac{d^2y}{dT^2} = -0.04y \frac{dy}{dt} = +$ concave up

(c) Find an expression for $y = f(t)$ by solving the differential equation $\underbrace{\frac{dy}{dt} = -0.02y^2}_{\text{Rate of change}}$ with the initial condition $f(0) = 10$.

$x = 0$
$y = 10$

$$\frac{\cancel{dT}}{y^2} \cdot \frac{dy}{\cancel{dT}} = -0.02\,\cancel{y^2} \cdot \frac{dT}{\cancel{y^2}}$$

$$\int y^{-2}\,dy = \int -0.02\,dT$$

$$\frac{1}{-1}\,y^{-2+1=-1} = -\frac{1}{y} = -0.02T + C$$

$$\frac{\cancel{y} \cdot \left(-\frac{1}{y}\right)}{\cancel{-0.02T+C}} = \frac{\cancel{(-0.02T+C)} \cdot y}{\cancel{-0.02T+C}}$$

$$\frac{-1}{-0.02T+C} = y$$

$$\frac{-1}{-0.02(0)+C} = 10$$

$$\frac{-1}{C} = 10 \Rightarrow \frac{-1}{10} = C$$

$$C = -0.1$$

$$F(T) = \frac{-1}{-0.02T - 0.1}$$

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

$$\frac{dy}{dT} = -0.02y^2 = \text{Always negative}$$

$$\frac{d^2y}{dT^2} = -0.04y \cdot \underbrace{\frac{dy}{dT}}_{\text{always negative}} = - \cdot + \cdot - = \text{positive}$$

amount of substance always +

$$\frac{d^2y}{dT^2} = -0.04y\,(-0.02y^2) = +0.0008\,y^3$$